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An improved on-line monitoring procedure for multiloop control systems

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Abstract

In this paper, an improved relay-based monitoring procedure is presented to on-line identify the maximum closed loop log modulus, $L_{c,max}$, for multiloop control systems. The proposed method addresses two potential problems inherent in previous methods: (a) too many relay tests are required; (b) the monitoring information fails to be exploited to redesign the controller when there is incentive to do so. Simulation results demonstrate the successful applications of the proposed on-line monitoring procedure as well as its utility in the redesign of the controllers.

Keywords: On-line monitoring procedure; Multiloop control systems; Fast Fourier transform

1. Introduction

Despite the developments and increasing sophistication in advanced controller designs in the past two decades, multiloop PID/PI controllers are still in widespread use in the process industries. This is because multiloop PID/PI controllers are easier to implement and cheaper to maintain as compared to other advanced multivariable control schemes. In addition, they are also known to be versatile for a wide range of processes and therefore are well accepted by operators. Based on the fact that PID/PI controllers will continue to dominate in process industries in the foreseeable future, any aspect that can lead to the effective use of this class of controllers will certainly bring about much economic benefit. This consideration is very important given the fact that many existing multiloop PID/PI controllers do not perform as well as expected. This is because most chemical processes are nonlinear in nature and process characteristics change by operating conditions and/or are difficult to identify precisely. Thus, the process model used for controller design may not give an accurate description of the actual process as the process dynamics vary with time. As a result, the performance of these controllers may dramatically deteriorate. Hence, it is an important task to have an efficient tool to determine on-line if the controller performance meets the design specification.

The goal of monitoring is to provide information which can be used to assess the current status of the controller and to assist the plant operators in deciding whether a redesign of the controller is necessary. However, most of the monitoring procedures reported in the literature are based on time domain techniques [1,2], and the deterioration of the control system cannot be determined until a significant deviation in the controlled variable is observed [3]. Recently, Chiang and Yu [3] and Ju and Chiu [4] proposed monitoring procedures based on relay tests to on-line identify the maximum closed loop log modulus [5], $L_{c,max}$, for SISO and MIMO systems, respectively. $L_{c,max}$ is considered because its value can be used to gauge the compromise made between performance and robustness of the controller. However, these previous results suffer two drawbacks. First, the monitoring procedures are tedious since the number of relay tests increases exponentially with the dimension of the system. Second, the information obtained from these monitoring procedures cannot be readily used in the redesign of the controller.

In this work, an alternative monitoring procedure is developed to address the aforementioned problems. The paper is organized as follows. Some basic concepts are introduced first, followed by the development of proposed monitoring procedure. Finally, one 3×3 linear system and one 2×2 nonlinear system are used to illustrate the proposed method and the conclusions are drawn.

2. Preliminaries

Relay feedback. Åström and Hägglund [6] introduced an autotuning procedure by using a relay inserted as a feedback controller as shown Fig. 1, where N denotes the relay. It

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Nomenclature

C_{A}	concentration of component A
C_p, C_{pc}	heat capacity of feed stream and coolant
E/R	activation energy
FFT	fast Fourier transform
G	process transfer function
h_{A}	heat transfer coefficient
K	controller transfer function
k_0	reaction rate constant
K_{p}	controller gain
L	number of the sampled points
$L_{\mathrm{c,max}}$	maximum closed loop log modulus
N	describing function of relay
q, q_{c}	feed and coolant flow-rate
T	temperature of the CSTR
T_{cf}, T_{f}	inlet coolant temperature and feed temperature
T_{s}	sampling interval
$\underline{\underline{U}}(\mathrm{j}\omega)$, $\underline{\underline{U_{\mathrm{s}}}}(\mathrm{j}\omega)$	frequency responses of $\underline{\underline{u}}(t)$ and $\underline{\underline{u_{\mathrm{s}}}}(t)$
$u(t)$, $\underline{u}(t)$, $\underline{\underline{u}}(t)$	signals of process inputs
$u_{\mathrm{s}}(t)$, $\underline{\underline{u_{\mathrm{s}}}}(t)$	the periodic stationary cycle part of $u(t)$ and $\underline{\underline{u}}(t)$
$\underline{\underline{V}}(\mathrm{j}\omega)$, $\underline{\underline{V_{\mathrm{s}}}}(\mathrm{j}\omega)$	frequency responses of $\underline{\underline{v}}(t)$ and $\underline{\underline{v_{\mathrm{s}}}}(t)$
$v(t)$, $\underline{v}(t)$, $\underline{\underline{v}}(t)$	signals of controller outputs
$v_{\mathrm{s}}(t)$, $\underline{\underline{v_{\mathrm{s}}}}(t)$	the periodic stationary cycle part of $v(t)$ and $\underline{\underline{v}}(t)$

Greek symbols

ΔH	heat of reaction
$\Delta\underline{\underline{U}}(\mathrm{j}\omega)$, $\Delta\underline{\underline{V}}(\mathrm{j}\omega)$	frequency responses of $\Delta\underline{\underline{u}}(t)$ and $\Delta\underline{\underline{v}}(t)$
$\Delta u(t)$, $\Delta\underline{\underline{u}}(t)$	transient part of $u(t)$ and $\underline{\underline{u}}(t)$
$\Delta v(t)$, $\Delta\underline{\underline{v}}(t)$	transient part of $v(t)$ and $\underline{\underline{v}}(t)$
ρ, ρ_{c}	density of reactor contents and coolant
τ_{c}	cycle period
τ_i	controller reset time
ω_i	frequency $(2\pi i/LT_{\mathrm{s}})$

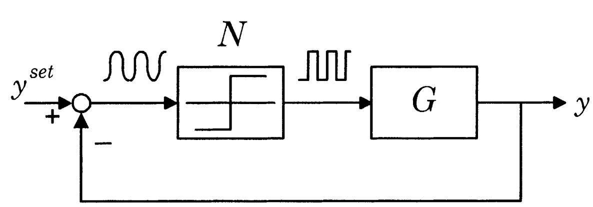

Fig. 1. Åström–Hägglund relay feedback system.

was reported that most of the process systems will exhibit stationary oscillation under relay feedback. Two types of relays are normally considered: an ideal relay and a relay with hysteresis.

Fast Fourier transform. Fast Fourier transform (FFT) is an efficient algorithm to transform time domain data into frequency domain. Suppose that the function $f(t)$ is absolutely integrable, i.e. $f(t)$ decays to zero in a finite time, or $f(mT_{\mathrm{s}}) = 0$ for $m \geq L$, where T_{s} is the sampling time, its frequency responses can be computed as [7]

$$F(\omega_i)=T_{\mathrm{s}}\sum_{m=0}^{L-1} f(mT_{\mathrm{s}})\,\mathrm{e}^{-\mathrm{j}\omega_i mT_{\mathrm{s}}}, \quad i = 0, 1, \ldots, L-1 \quad (1)$$

where the frequency ω_i is defined as

$$\omega_i = \frac{2\pi i}{LT_{\mathrm{s}}}, \quad i = 0, 1, \ldots, L-1 \quad (2)$$

$L_{\mathrm{c,max}}$. The objective of the proposed monitoring procedure is to monitor controller performance by on-line identification of the maximum closed loop log modulus, $L_{\mathrm{c,max}}$ [5]. For multivariable feedback control systems, $L_{\mathrm{c,max}}$ is defined as

$$L_{\mathrm{c,max}} = 20\log\left(\max_{\omega}\left|\frac{-1+\det(I+GK)}{\det(I+GK)}\right|\right) \quad (3)$$

where G and K denote the process and controller transfer function, respectively.

From its definition, $L_{\mathrm{c,max}}$ measures the closeness of the curve GK to the critical point $(-1, 0)$ in the Nyquist plot. Therefore $L_{\mathrm{c,max}}$ can be used to measure the robustness of the control systems. A large $L_{\mathrm{c,max}}$ means that the control systems can tolerate a small amount of uncertainties. Thus an increasing trend in the $L_{\mathrm{c,max}}$ will indicate the proximity to the closed loop instability. Therefore, retuning of controller is then necessary to maintain stability. Luyben [8] proposed the BLT tuning rule $L_{\mathrm{c,max}} = 2n$ for the $n \times n$ multiloop control systems.

3. An improved monitoring procedure for multiloop systems

To on-line monitor $L_{\mathrm{c,max}}$ for an $n \times n$ multiloop control system, Ju and Chiu [4] proposed a monitoring scheme as depicted in Fig. 2. In each control loop, a selector is placed between the controller and process so that the control loop can be put either on control mode (denoted by c) or monitoring mode (denoted by m). Subsequently, $L_{\mathrm{c,max}}$ can be calculated on-line from the information obtained in the relay tests [4]. However, the monitoring procedure in [4] is tedious since the number of relay tests increases exponentially with the dimension of the system. To simplify the monitoring procedure in [4], this paper presents an alternative monitoring procedure in which n relays are placed simultaneously between the controllers and process in all

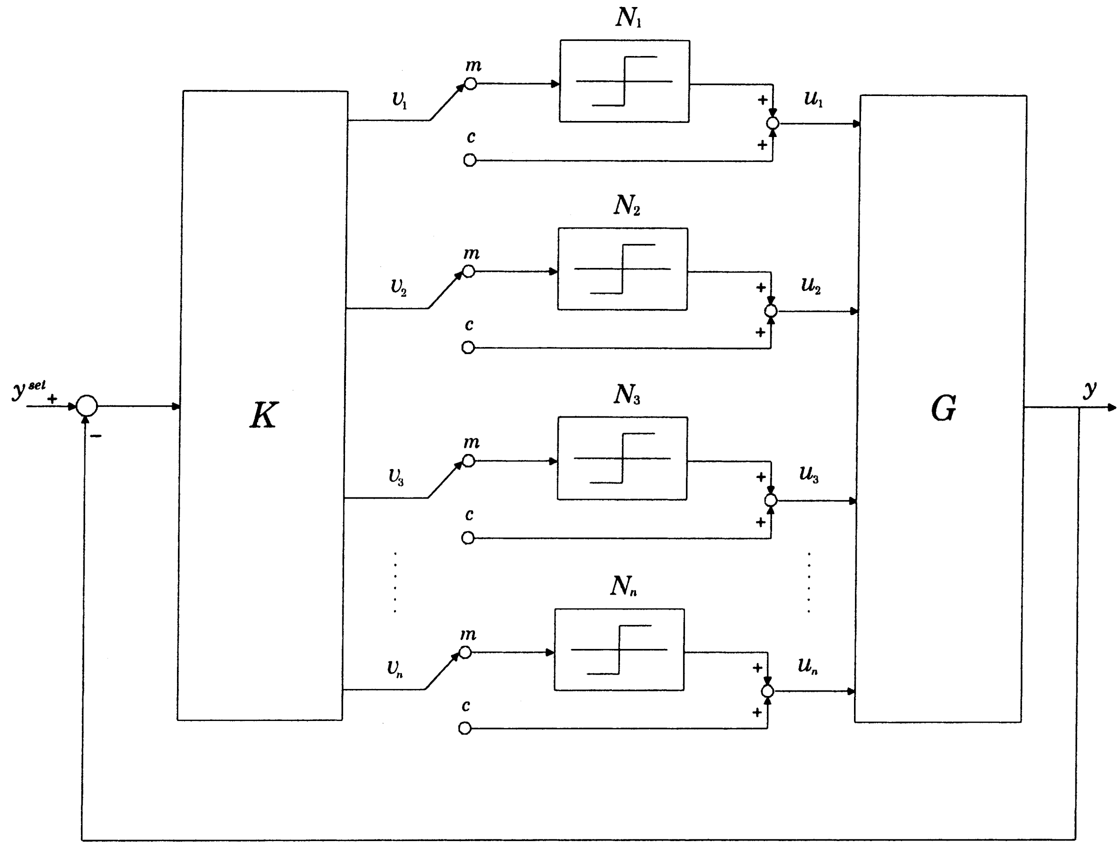

Fig. 2. Monitoring scheme for an $n \times n$ multiloop control system.

control loops, i.e. n control loops are in monitoring mode in Fig. 2. In order to calculate $L_{\mathrm{c,max}}$ on-line, the aforementioned relay test needs to be conducted n times by using n relays with different hysteresis. An immediate benefit of the proposed method is that the number of relay tests required in the monitoring procedure is significantly smaller than that in [4], e.g. the proposed method needs two relay tests as compared to nine tests in [4] for a 2×2 system. Finally, from the system responses obtained during the monitoring procedure, $L_{\mathrm{c,max}}$ can be computed using Eqs. (15) and (16) derived in the ensuing development of this section.

In what follows, the analysis of the proposed monitoring procedure is given. For each relay test in the proposed procedure, the relay input $\underline{v}^i(t)$ and relay output $\underline{u}^i(t)$, where the superscript i ($i = 1, 2, \ldots, n$) denotes the serial number of the relay test, are recorded until stationary oscillation is reached. After n relay tests are conducted, the following two matrices can be set up:

$$\underline{\underline{v}}(t) = \begin{bmatrix} \underline{v}^1(t) & \underline{v}^2(t) & \ldots & \underline{v}^n(t) \end{bmatrix} = \begin{bmatrix} v_1^1(t) & v_1^2(t) & \cdots & v_1^n(t) \\ v_2^1(t) & v_2^2(t) & \cdots & v_2^n(t) \\ \vdots & \vdots & \ddots & \vdots \\ v_n^1(t) & v_n^2(t) & \cdots & v_n^n(t) \end{bmatrix} \tag{4}$$

$$\underline{\underline{u}}(t) = \begin{bmatrix} \underline{u}^1(t) & \underline{u}^2(t) & \ldots & \underline{u}^n(t) \end{bmatrix} = \begin{bmatrix} u_1^1(t) & u_1^2(t) & \cdots & u_1^n(t) \\ u_2^1(t) & u_2^2(t) & \cdots & u_2^n(t) \\ \vdots & \vdots & \ddots & \vdots \\ u_n^1(t) & u_n^2(t) & \cdots & u_n^n(t) \end{bmatrix} \tag{5}$$

where the subscript of the elements in $\underline{\underline{v}}(t)$ and $\underline{\underline{u}}(t)$ is the index of control loops. For example, $v_1^2(t)$ is the relay input in the control loop 1 of the second relay test.

From the monitoring scheme in Fig. 2, the following equation holds:

$$K(s)G(s) = -\underline{\underline{V}}(s)\underline{\underline{U}}(s)^{-1} \tag{6}$$

where $\underline{\underline{V}}(s)$ and $\underline{\underline{U}}(s)$ are the Laplace transforms of $\underline{\underline{v}}(t)$ and $\underline{\underline{u}}(t)$. In frequency domain, Eq. (6) is written as

$$K(\mathrm{j}\omega)G(\mathrm{j}\omega) = -\underline{\underline{V}}(\mathrm{j}\omega)\underline{\underline{U}}(\mathrm{j}\omega)^{-1} \tag{7}$$

Since both relay inputs $\underline{\underline{v}}(t)$ and relay outputs $\underline{\underline{u}}(t)$ are not absolutely integrable, they cannot be transformed to frequency response by directly using FFT. However, since both signals reach to stationary oscillation, they can be decomposed as follows [9]:

$$\underline{\underline{v}}(t) = \Delta\underline{\underline{v}}(t) + \underline{\underline{v_\mathrm{s}}}(t) = \begin{bmatrix} \Delta v_1^1(t) & \Delta v_1^2(t) & \cdots & \Delta v_1^n(t) \\ \Delta v_2^1(t) & \Delta v_2^2(t) & \cdots & \Delta v_2^n(t) \\ \vdots & \vdots & \ddots & \vdots \\ \Delta v_n^1(t) & \Delta v_n^2(t) & \cdots & \Delta v_n^n(t) \end{bmatrix} + \begin{bmatrix} v_{1\mathrm{s}}^1(t) & v_{1\mathrm{s}}^2(t) & \cdots & v_{1\mathrm{s}}^n(t) \\ v_{2\mathrm{s}}^1(t) & v_{2\mathrm{s}}^2(t) & \cdots & v_{2\mathrm{s}}^n(t) \\ \vdots & \vdots & \ddots & \vdots \\ v_{n\mathrm{s}}^1(t) & v_{n\mathrm{s}}^2(t) & \cdots & v_{n\mathrm{s}}^n(t) \end{bmatrix} \tag{8}$$

$$\underline{\underline{u}}(t) = \Delta\underline{\underline{u}}(t) + \underline{\underline{u_\mathrm{s}}}(t) = \begin{bmatrix} \Delta u_1^1(t) & \Delta u_1^2(t) & \cdots & \Delta u_1^n(t) \\ \Delta u_2^1(t) & \Delta u_2^2(t) & \cdots & \Delta u_2^n(t) \\ \vdots & \vdots & \ddots & \vdots \\ \Delta u_n^1(t) & \Delta u_n^2(t) & \cdots & \Delta u_n^n(t) \end{bmatrix} + \begin{bmatrix} u_{1\mathrm{s}}^1(t) & u_{1\mathrm{s}}^2(t) & \cdots & u_{1\mathrm{s}}^n(t) \\ u_{2\mathrm{s}}^1(t) & u_{2\mathrm{s}}^2(t) & \cdots & u_{2\mathrm{s}}^n(t) \\ \vdots & \vdots & \ddots & \vdots \\ u_{n\mathrm{s}}^1(t) & u_{n\mathrm{s}}^2(t) & \cdots & u_{n\mathrm{s}}^n(t) \end{bmatrix} \tag{9}$$

where $\underline{\underline{v_\mathrm{s}}}(t)$ or $\underline{\underline{u_\mathrm{s}}}(t)$ are the periodic stationary responses of $\underline{\underline{v}}(t)$ and $\underline{\underline{u}}(t)$, respectively. Since both $\Delta\underline{\underline{v}}(t)$ and $\Delta\underline{\underline{u}}(t)$ eventually decay to zero, their frequency responses can be obtained with the standard FFT technique, i.e. at each frequency ω_i ($i = 0$ to $L-1$):

$$\Delta\underline{\underline{V}}(\mathrm{j}\omega_i) = T_\mathrm{s} \begin{bmatrix} \sum_{m=0}^{L-1}\Delta v_1^1(mT_\mathrm{s})\,\mathrm{e}^{-\mathrm{j}\omega_i mT_\mathrm{s}} & \sum_{m=0}^{L-1}\Delta v_1^2(mT_\mathrm{s})\,\mathrm{e}^{-\mathrm{j}\omega_i mT_\mathrm{s}} & \cdots & \sum_{m=0}^{L-1}\Delta v_1^n(mT_\mathrm{s})\,\mathrm{e}^{-\mathrm{j}\omega_i mT_\mathrm{s}} \\ \sum_{m=0}^{L-1}\Delta v_2^1(mT_\mathrm{s})\,\mathrm{e}^{-\mathrm{j}\omega_i mT_\mathrm{s}} & \sum_{m=0}^{L-1}\Delta v_2^2(mT_\mathrm{s})\,\mathrm{e}^{-\mathrm{j}\omega_i mT_\mathrm{s}} & \cdots & \sum_{m=0}^{L-1}\Delta v_2^n(mT_\mathrm{s})\,\mathrm{e}^{-\mathrm{j}\omega_i mT_\mathrm{s}} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{m=0}^{L-1}\Delta v_n^1(mT_\mathrm{s})\,\mathrm{e}^{-\mathrm{j}\omega_i mT_\mathrm{s}} & \sum_{m=0}^{L-1}\Delta v_n^2(mT_\mathrm{s})\,\mathrm{e}^{-\mathrm{j}\omega_i mT_\mathrm{s}} & \cdots & \sum_{m=0}^{L-1}\Delta v_n^n(mT_\mathrm{s})\,\mathrm{e}^{-\mathrm{j}\omega_i mT_\mathrm{s}} \end{bmatrix} \tag{10}$$

$$\Delta\underline{\underline{U}}(\mathrm{j}\omega_i) = T_\mathrm{s} \begin{bmatrix} \sum_{m=0}^{L-1}\Delta u_1^1(mT_\mathrm{s})\,\mathrm{e}^{-\mathrm{j}\omega_i mT_\mathrm{s}} & \sum_{m=0}^{L-1}\Delta u_1^2(mT_\mathrm{s})\,\mathrm{e}^{-\mathrm{j}\omega_i mT_\mathrm{s}} & \cdots & \sum_{m=0}^{L-1}\Delta u_1^n(mT_\mathrm{s})\,\mathrm{e}^{-\mathrm{j}\omega_i mT_\mathrm{s}} \\ \sum_{m=0}^{L-1}\Delta u_2^1(mT_\mathrm{s})\,\mathrm{e}^{-\mathrm{j}\omega_i mT_\mathrm{s}} & \sum_{m=0}^{L-1}\Delta u_2^2(mT_\mathrm{s})\,\mathrm{e}^{-\mathrm{j}\omega_i mT_\mathrm{s}} & \cdots & \sum_{m=0}^{L-1}\Delta u_2^n(mT_\mathrm{s})\,\mathrm{e}^{-\mathrm{j}\omega_i mT_\mathrm{s}} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{m=0}^{L-1}\Delta u_n^1(mT_\mathrm{s})\,\mathrm{e}^{-\mathrm{j}\omega_i mT_\mathrm{s}} & \sum_{m=0}^{L-1}\Delta u_n^2(mT_\mathrm{s})\,\mathrm{e}^{-\mathrm{j}\omega_i mT_\mathrm{s}} & \cdots & \sum_{m=0}^{L-1}\Delta u_n^n(mT_\mathrm{s})\,\mathrm{e}^{-\mathrm{j}\omega_i mT_\mathrm{s}} \end{bmatrix} \tag{11}$$

For a periodic function, the next equation [10] can be used to compute its frequency response:

$$F_\mathrm{p}(s) = \frac{1}{1-\mathrm{e}^{-s\tau_\mathrm{c}}}\int_0^{\tau_\mathrm{c}} f_\mathrm{p}(t)\,\mathrm{e}^{-st}\,\mathrm{d}t \tag{12}$$

where $F_\mathrm{p}(s)$ denotes the Laplace transform of a periodic function $f_\mathrm{p}(t)$ with period τ_c, i.e. $f_\mathrm{p}(t) = f_\mathrm{p}(t+\tau_\mathrm{c})\ \forall t$.

By using Eq. (12), the frequency responses of $\underline{\underline{v_\mathrm{s}}}(t)$ or $\underline{\underline{u_\mathrm{s}}}(t)$ are obtained as follows:

$$\underline{\underline{V_s}}(j\omega_i) = \begin{bmatrix} \frac{T_s}{1-e^{-j\omega_i\tau_{c1}^1}}\sum_{m=0}^{N_1^1} v_{1s}^1(mT_s)\,e^{-j\omega_i mT_s} & \frac{T_s}{1-e^{-j\omega_i\tau_{c1}^2}}\sum_{m=0}^{N_1^2} v_{1s}^2(mT_s)\,e^{-j\omega_i mT_s} & \cdots & \frac{T_s}{1-e^{-j\omega_i\tau_{c1}^n}}\sum_{m=0}^{N_1^n} v_{1s}^n(mT_s)\,e^{-j\omega_i mT_s} \\ \frac{T_s}{1-e^{-j\omega_i\tau_{c2}^1}}\sum_{m=0}^{N_2^1} v_{2s}^1(mT_s)\,e^{-j\omega_i mT_s} & \frac{T_s}{1-e^{-j\omega_i\tau_{c2}^1}}\sum_{m=0}^{N_2^2} v_{2s}^2(mT_s)\,e^{-j\omega_i mT_s} & \cdots & \frac{T_s}{1-e^{-j\omega_i\tau_{c2}^n}}\sum_{m=0}^{N_2^n} v_{2s}^n(mT_s)\,e^{-j\omega_i mT_s} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{T_s}{1-e^{-j\omega_i\tau_{cn}^1}}\sum_{m=0}^{N_n^1} v_{ns}^1(mT_s)\,e^{-j\omega_i mT_s} & \frac{T_s}{1-e^{-j\omega_i\tau_{cn}^2}}\sum_{m=0}^{N_n^2} v_{ns}^2(mT_s)\,e^{-j\omega_i mT_s} & \cdots & \frac{T_s}{1-e^{-j\omega_i\tau_{cn}^n}}\sum_{m=0}^{N_n^n} v_{ns}^n(mT_s)\,e^{-j\omega_i mT_s} \end{bmatrix} \quad (13)$$

$$\underline{\underline{U_s}}(j\omega_i) = \begin{bmatrix} \frac{T_s}{1-e^{-j\omega_i\tau_{c1}^1}}\sum_{m=0}^{N_1^1} u_{1s}^1(mT_s)\,e^{-j\omega_i mT_s} & \frac{T_s}{1-e^{-j\omega_i\tau_{c1}^2}}\sum_{m=0}^{N_1^2} u_{1s}^2(mT_s)\,e^{-j\omega_i mT_s} & \cdots & \frac{T_s}{1-e^{-j\omega_i\tau_{c1}^n}}\sum_{m=0}^{N_1^n} u_{1s}^n(mT_s)\,e^{-j\omega_i mT_s} \\ \frac{T_s}{1-e^{-j\omega_i\tau_{c2}^1}}\sum_{m=0}^{N_2^1} u_{2s}^1(mT_s)\,e^{-j\omega_i mT_s} & \frac{T_s}{1-e^{-j\omega_i\tau_{c2}^2}}\sum_{m=0}^{N_2^2} u_{2s}^2(mT_s)\,e^{-j\omega_i mT_s} & \cdots & \frac{T_s}{1-e^{-j\omega_i\tau_{c2}^n}}\sum_{m=0}^{N_2^n} u_{2s}^n(mT_s)\,e^{-j\omega_i mT_s} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{T_s}{1-e^{-j\omega_i\tau_{cn}^1}}\sum_{m=0}^{N_n^1} u_{ns}^1(mT_s)\,e^{-j\omega_i mT_s} & \frac{T_s}{1-e^{-j\omega_i\tau_{cn}^2}}\sum_{m=0}^{N_n^2} u_{ns}^2(mT_s)\,e^{-j\omega_i mT_s} & \cdots & \frac{T_s}{1-e^{-j\omega_i\tau_{cn}^n}}\sum_{m=0}^{N_n^n} u_{ns}^n(mT_s)\,e^{-j\omega_i mT_s} \end{bmatrix} \quad (14)$$

where $N_k^l = (\tau_{ck}^l - T_s)/T_s$ $(k, l = 1 \sim n)$ and τ_{ck}^l stands for the period of stationary oscillations in the kth control loop of the lth relay test.

Based on the preceding discussion, at each frequency ω_i, Eq. (7) can be computed as

$$K(j\omega_i)G(j\omega_i) = -(\underline{\underline{V_s}}(j\omega_i) + \Delta\underline{\underline{V}}(j\omega_i))(\underline{\underline{U_s}}(j\omega_i) + \Delta\underline{\underline{U}}(j\omega_i))^{-1} \quad (15)$$

where $\underline{\underline{V_s}}(jw_i)$, $\Delta\underline{\underline{V}}(j\omega_i)$, $\underline{\underline{U_s}}(j\omega_i)$ and $\Delta\underline{\underline{U}}(j\omega_i)$ are obtained from Eqs. (13), (10), (14) and (11), respectively. Consequently, the frequency response of $K(jw_i)G(j\omega_i)$ in the frequency interval $[\,0 \quad 2\pi(L-1)/LT_s\,]$ can be obtained. Finally, to calculate $L_{c,\max}$, one notes that Eq. (3) can be rewritten as

$$L_{c,\max} = 20\log\left(\max_{\omega}\left|\frac{-1+\det(I+KG)}{\det(I+KG)}\right|\right) \quad (16)$$

Since the frequency response of KG is known, $L_{c,\max}$ is readily computed from Eq. (16).

4. Examples

4.1. Linear system

To test the proposed monitoring procedure, a 3 × 3 multiloop control system studied in [11] is considered, where

$$G(s) = \begin{bmatrix} \frac{0.66\,e^{-2.6s}}{6.7s+1} & \frac{-0.61\,e^{-3.5s}}{8.64s+1} & \frac{-0.0049\,e^{-s}}{9.06s+1} \\ \frac{1.11\,e^{-6.5s}}{3.25s+1} & \frac{-2.36\,e^{-3s}}{5s+1} & \frac{-0.01\,e^{-1.2s}}{7.09s+1} \\ \frac{-34.68\,e^{-9.2s}}{8.15s+1} & \frac{46.2\,e^{-9.4s}}{10.9s+1} & \frac{0.87(11.61s+1)\,e^{-s}}{(3.89s+1)(18.8s+1)} \end{bmatrix} \quad (17)$$

$$K(s) = \begin{bmatrix} 1.509\left(1+\frac{1}{35.26s}\right) & 0 & 0 \\ 0 & -0.295\left(1+\frac{1}{38.7s}\right) & 0 \\ 0 & 0 & 2.629\left(1+\frac{1}{14.211s}\right) \end{bmatrix} \quad (18)$$

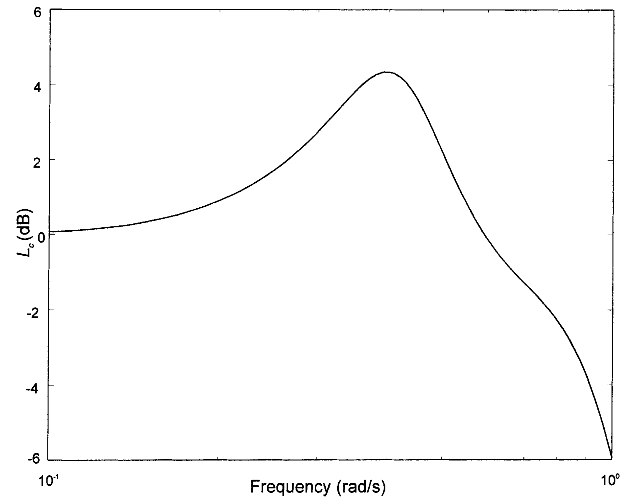

Fig. 3. Closed loop log modulus versus frequency.

In the proposed monitoring procedure, three relay tests are performed. The identified $L_{c,max}$ is 4.357 dB as illustrated in Fig. 3. It differs from the actual value of 4.342 dB by 0.34%. In comparison, the monitoring procedure by Ju and Chiu [4] requires overwhelming 21 relay tests. Thus, it is clear that the proposed method is more effective than the previous method without sacrificing the accuracy in the on-line identification of $L_{c,max}$.

Another advantage of the proposed method is that the information obtained from the monitoring procedure can be readily used in the retuning of the controller. Since both the frequency response of $K(s)G(s)$ and the controller's parameters are known, the frequency response of the current process dynamics can be obtained as

$$G(j\omega) = K(j\omega)^{-1}K(j\omega)G(j\omega) \quad (19)$$

With this updated information of the process, the controller can be redesigned if there is an incentive to do so. For example, according to the BLT criterion [8], a reasonable value of $L_{c,max}$ for a 3 × 3 system is 6 dB. To achieve this, a "retuning" factor β is first introduced such that

$$K^*(s) = \frac{1}{\beta}\begin{bmatrix} 1.509\left(1+\frac{1}{\beta}\frac{1}{35.26s}\right) & 0 & 0 \\ 0 & -0.295\left(1+\frac{1}{\beta}\frac{1}{38.7s}\right) & 0 \\ 0 & 0 & 2.629\left(1+\frac{1}{\beta}\frac{1}{14.211s}\right) \end{bmatrix} \quad (20)$$

Next, with the identified G obtained from Eq. (19), $L_{c,max}$ can be adjusted on-line to be equal to 6 dB by tuning β. The resulting β from this procedure is 0.871 as compared to 0.869 obtained when the process is known exactly. Finally, it is worth noting that, unlike the detuning factor used in the BLT method, β is allowed to be smaller than 1 as shown above. As such, β is more appropriately termed as "retuning" factor in the context discussed in this paper.

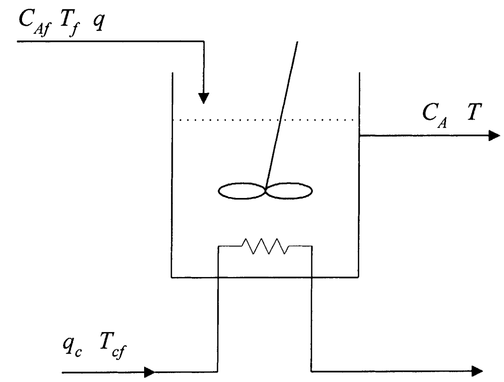

Fig. 4. The continuous stirred tank reactor.

4.2. Nonlinear system

The 2 × 2 nonlinear system under consideration is a continuous stirred tank reactor (CSTR) shown in Fig. 4. A single irreversible, exothermic reaction A → B is assumed to occur in the reactor. The process model consists of two nonlinear ordinary differential equations [12],

$$\dot{C}_A = \frac{q}{100}(C_{Af} - C_A) - k_0 C_A\, e^{-E/RT} \quad (21)$$

$$\dot{T} = \frac{q}{100}(T_f - T) + \frac{(-\Delta H)k_0 C_A}{\rho C_p} e^{-E/RT} + \frac{\rho_c C_{pc}}{100\rho C_p} q_c[1 - e^{-h_A/(q_c\rho_c C_{pc})}] \times (T_{cf} - T) \quad (22)$$

where C_A is the effluent concentration of component A, T the reactor temperature, C_{Af} the feed composition, and q_c the coolant flow-rate. The remaining model parameters are defined in the Nomenclature and the nominal operating conditions are given in Table 1 . A 2 × 2 multiloop PI controller is designed to control the effluent concentration at set-point $C_A^{set} = 0.1$ mol/l and the temperature at set-point $T^{set} = 438.54$ K by manipulating q_c and C_{Af}, respectively.

Table 1
Nominal CSTR operating conditions

$q = 100$ l/min	$E/R = 1 \times 10^4$ K
$C_{Af} = 1$ mol/l	$-\Delta H = 2 \times 10^5$ cal/mol
$T_f = 350$ K	$\rho, \rho_c = 1000$ g/l
$T_{cf} = 350$ K	$C_p, C_{pc} = 1$ cal/(g K)
$h_A = 7 \times 10^5$ cal/(min K)	$q_c = 103.41$ l/min (operating condition 1)
$k_0 = 7.2 \times 10^{10}$ min^{-1}	$q_c = 108.10$ l/min (operating condition 2)

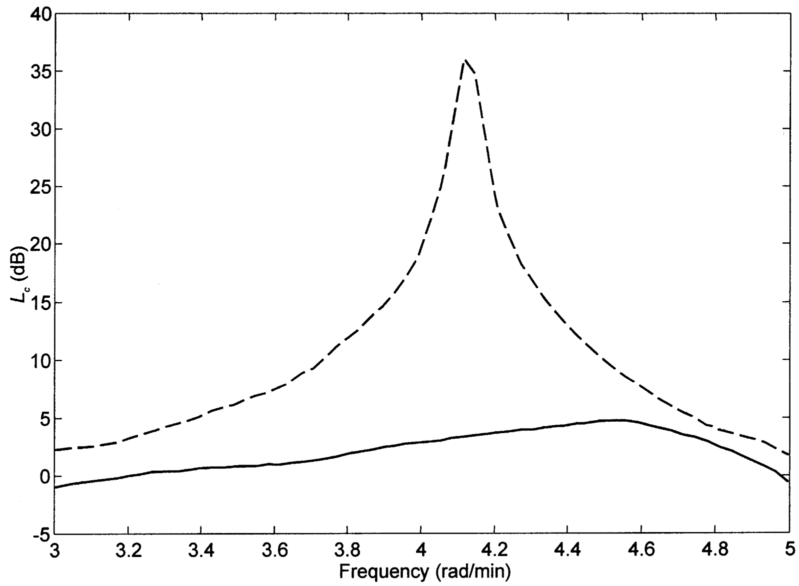

Fig. 5. Closed loop log modulus versus frequency for the CSTR example: solid line — $C_A^{set} = 0.1\,mol/l$, $T^{set} = 438.54\,K$; dashed line — $C_A^{set} = 0.12\,mol/l$, $T^{set} = 434.64\,K$.

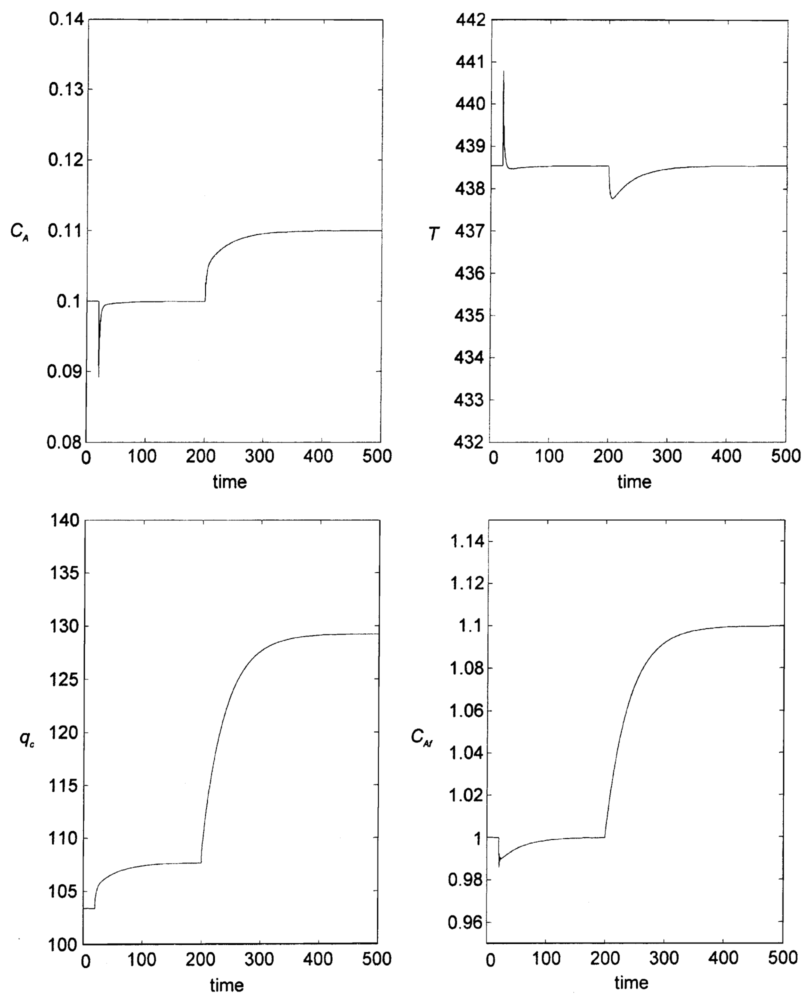

Fig. 6. Closed loop responses at operating condition, $C_A^{set} = 0.1\,mol/l$ and $T^{set} = 438.54\,K$.

The parameters of the PI controller in the concentration loop are $K_c = 50$, $\tau_i = 0.5$ and those in the temperature loop are $K_c = 0.0054$, $\tau_i = 1.92$. The measurement time delays of concentration loop and temperature loop are 0.1 and 0.05 min, respectively.

To illustrate the application of the proposed monitoring procedure, the robustness of the multiloop PI controller at two different operating conditions is assessed from the comparison of the identified $L_{c,\max}$. For the aforementioned operating condition, the $L_{c,\max}$ is equal to 4.74 dB which is the peak value of solid line in Fig. 5. Since the recommended $L_{c,\max}$ value is 4 dB for this 2×2 system, it implies that the control system is quite robust at this operating point. Assume that the PI controller remains unchanged, but the operating condition is changed to $C_A^{set} = 0.12$ mol/l and $T^{set} = 434.64$ K. Based on the same monitoring procedure, the $L_{c,\max}$ is found to increase to 36.03 dB (dashed line in Fig. 5), which means that the stability margin of the control system is significantly smaller than that in previous operating condition. Consequently, an oscillatory or even unstable response is likely to occur if disturbances enter into the control system or a set-point change is made. Simulation results in Figs. 6 and 7 confirm this observation by comparing the closed loop responses when the coolant temperature changes from 350 to 353.5 K at $t = 20$, and subsequently a 10% set-point change in composition loop occurs at $t = 200$, at two respective operating conditions. It is clear that the controller gives much better servo and regulatory responses at original operating condition, namely $C_A^{set} = 0.1$ mol/l and $T^{set} = 438.54$ K. In comparison, the control system exhibits a highly oscillatory response when set-point change is made at the second operating condition.

Since the multiloop PI controller is designed with respect to the original operating condition, it is expected to perform satisfactorily in the vicinity of $C_A^{set} = 0.1$ mol/l and $T^{set} = 438.54$ K. Thus, it is not surprising that a large $L_{c,\max}$ results from the changes in the operating conditions. This suggests a need to retune controller parameters for better performance

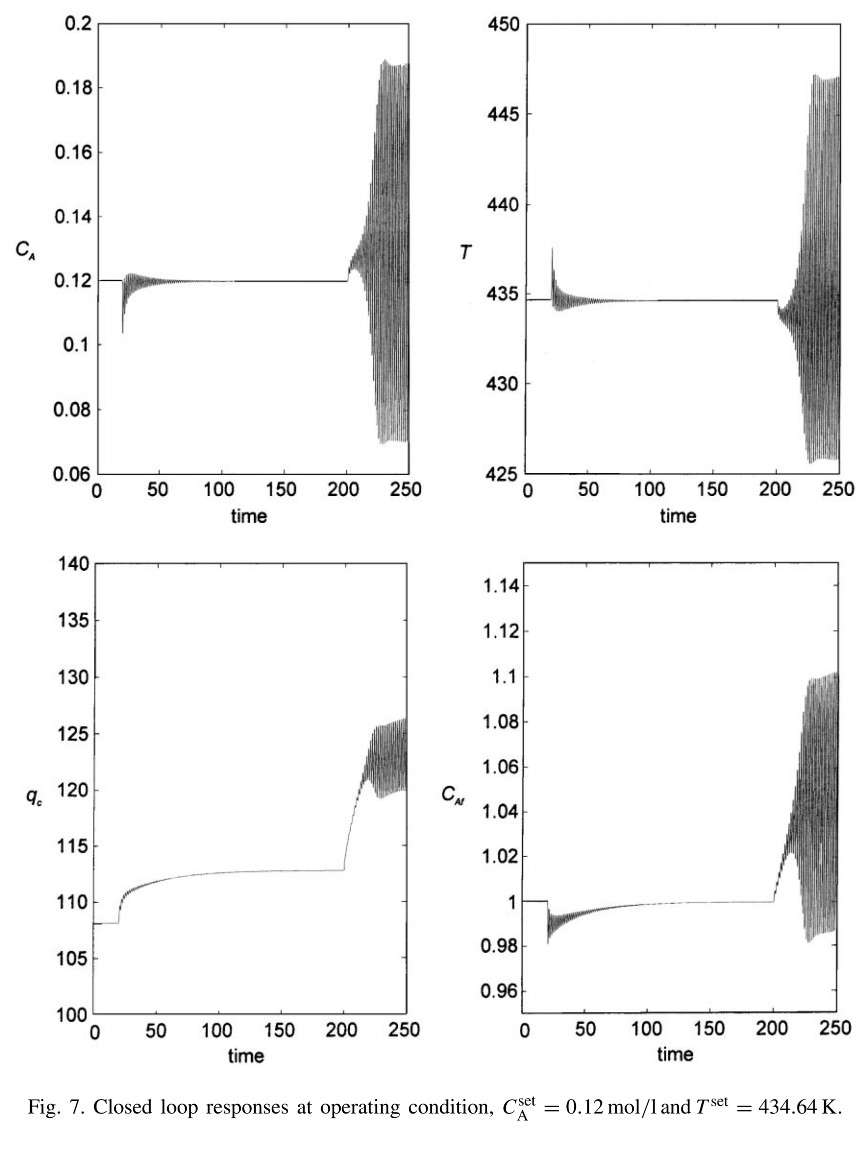

Fig. 7. Closed loop responses at operating condition, $C_A^{set} = 0.12$ mol/l and $T^{set} = 434.64$ K.

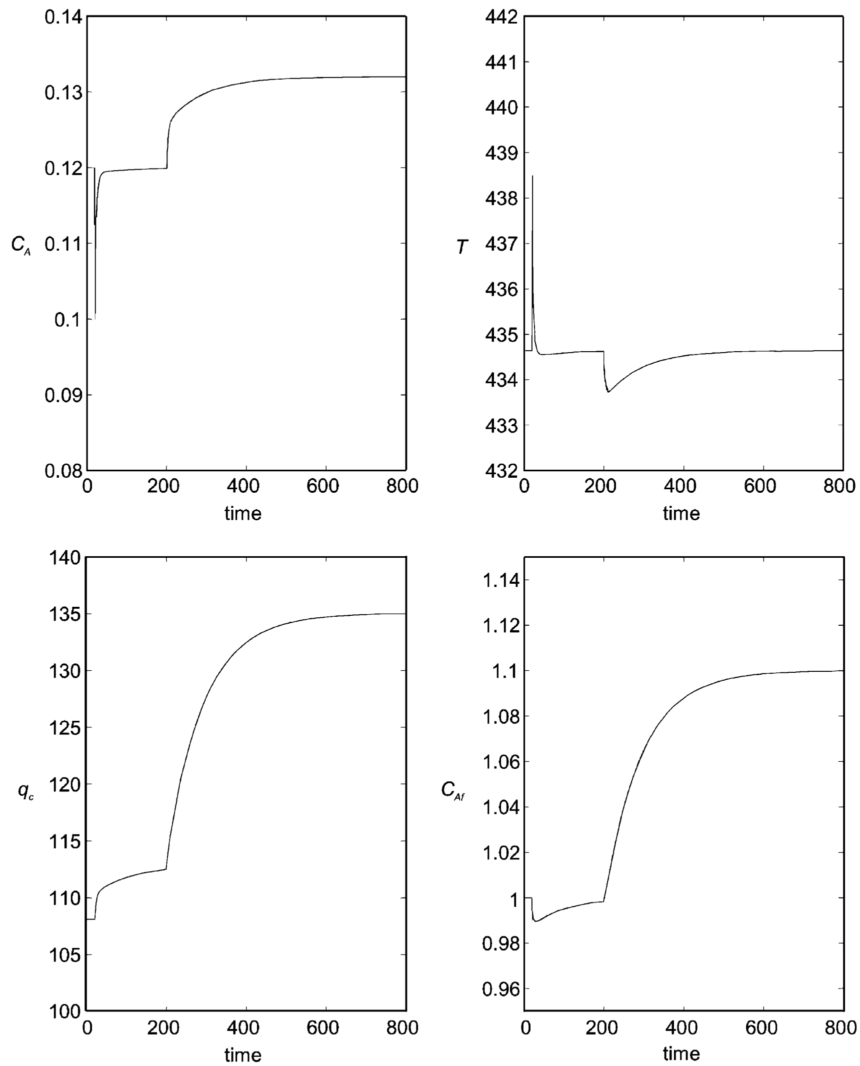

Fig. 8. Closed loop responses (after retuning of PI controller) at operating condition, $C_A^{set} = 0.12$ mol/l and $T^{set} = 434.64$ K.

at the second operating condition. Using the same on-line retuning procedure as discussed before, the retuning factor β is found to be 2.62. Under the same simulation conditions, the servo and regulatory responses of this new controller are illustrated in Fig. 8. Evidently, a marked improvement over the original controller design at the second operating condition (Fig. 7) is achieved.

5. Conclusions

An improved relay-based monitoring procedure to identify $L_{c,max}$ on-line is proposed. Compared to the previous methods the testing time is greatly shortened since fewer relay tests are needed. Furthermore, it allows one to adjust controller parameters on-line by using the information obtained from the monitoring procedure. Literature examples are used to demonstrate the successful applications of the proposed monitoring procedure as well as its utility in the redesign of the controllers.

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